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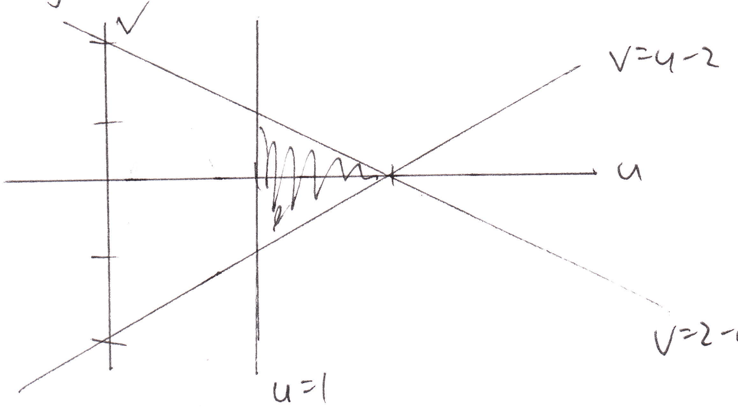
Bergman '96 Final Exam

#4 Let R be the triangle in the (u,v) plane defined by the inequalities $u \geq 1$, $u-2 \leq v \leq 2-u$. Let T be the image of R in the (x,y) plane under the transformation given by $x=u^2-v^2$, $y=uv$. Express the double integral $\iint_T e^{x^2-y^2}\,dA$ as an iterated integral, in the variable u and v.



$$x=u^2-v^2$$
$$y=uv$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix} = 2u^2+2v^2$$

to get x^2-y^2 must square both x and y

$$x^2=(u^2-v^2)^2 = u^4-2u^2v^2+v^4$$
$$y^2=(uv)^2 = u^2v^2$$

$x^2-y^2 = u^4-3u^2v^2+v^4$

substitute

$$\iint_T e^{x^2-y^2}\,dA = \int_1^2 \int_{u-2}^{2-u} e^{u^4-3u^2v^2+v^4}\,dv\,du$$